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Research Paper

Multi-Attribute Decision Making Approach Based on Neutral Membership Degree of Picture Fuzzy Set

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Abstract: In this study we proposed a new weighted aggregation operator for ranking the picture fuzzy numbers (PFNs) which is based on neutral membership value of PFN. As the picture fuzzy set (PFS) is an extension version of intuitionistic fuzzy set theory with introducing the neutral membership value during data analysis. The neutral membership value in PFS reflecting the ambiguous nature of the subject to judgment. The ambiguity is depending on the neutral membership value of PFN. The proposed weighted aggregation operator manages the ambiguity according to neutral membership value. Then, the aggregation operator applies in a multi attribute decision making method where attribute value of the alternative are picture fuzzy numbers. In the decision making process, the weight of attributes are calculated according to neutral values and aggregate the multiple attributes into a single PFN. Then estimate the individual score value of the alternatives. Lastly, ranking the alternative according to score value. Finally, a practical example for students' performance in the multiple paper examination is highlighted for verifying the developed approach and demonstrates its practicality and effectiveness.

Keywords: Aggregation operators, Decision-making, Picture fuzzy set, Weighted Aggregation operators.

1. Introduction

Fuzzy set(FS) was introduced by Zadesh [1] in 1965, then numerous serious researchers are extended the concept of FS in different way depending on the different situations and problems types such as type-2 fuzzy set(T2FS) [19, 28], rough set(RS) [16],Vague sets [20] fuzzy soft set(FSS) [17], rough soft set [22],Vague soft sets [20], interval value fuzzy set [23], hesitant fuzzy set [24], intuitionistic fuzzy set(IFS) [21], intuitionistic fuzzy soft set [18] etc. Among those extension versions of fuzzy set have some advantage as well as limitation based on the level of fuzziness, uncertainty, impression etc. to manage those limitation of the extension fuzzy set, further extended with adding some new concepts. The IFS is the extension version of FS with considering the negative membership value of elements. Recently, the IFS is further extended into the PFS(Picture Fuzzy Set) with adding another membership value with positive and negative value of the elements [26, 27]. The new membership value called neutral membership value which is indicated the neutral part of the elements. The word "picture" in the PFS refers to generality of this set is the direct extension of FS and IFS. The extension fuzzy set manage the imprecision, vagueness, fuzziness and uncertainty of the problem efficiently and easily. Parallelly, the researchers were deeply focused in this area for further updating and proposed number of operator and operation those are successfully applied in the different field like decision making, clustering, medical diagnosis, pattern recognition and optimization [5]. The aggregation operator merges the performance of all criteria for the alternative that can reduce the rapidly growing complexity and size of the problems. Also, the weighted and ordered weighted aggregation operator fuses some useless information during data analysis. For this purpose, Xu and Yagar[8] represent a geometric aggregation operator while Xu[9] presented a weighted averaging operator for aggregating the different intuitionistic fuzzy numbers(IFNs). S Das el al. [6] proposed intuitionistic multi-fuzzy weighted averaging (IMFWA) operator merging number of intuitionistic multi-fuzzy numbers into single one. H. Gang [7] proposed different type of aggregation operators for picture fuzzy numbers. Moreover, a lot researchers are sincerely concentrated on this topic and developed different type aggregation operator apply in MCDM procedure to manage the different situation of decision making problem. Those decision making problem has the varieties criteria's and constraints to generate the accepted solution. Some are the common MCDM procedure are ELECTRE [25], TOPSIS [11, 15], AHP [14], VIKOR [10, 29] and TODIM [4].

We are observed from the previous discussion that the aggregation operations are used in FS, IFS, IMPS, IVFS or PFS set theories. Those proposed aggregation operators are successfully applied in our real life problem. There are different parts within the data elements of the set theories like positive and negative membership value in IFS or positive, negative and neutral membership value of PFS etc. Then the importance of the membership values are same in the aggregation operators whereas the neutral membership values in the PFS have a special characteristic. This may be positive or may negative, this membership value represent the uncertainty of uncertainty. If any one of the PFS with zero neutral membership value then the PFS became IFS and we sure the positiveness and negativeness accurately but with some specific value of neutral membership value introduce the uncertainty. That mean positiveness and negativeness depend on the positive and negative membership values respectively and neutral membership value indicate the undecided part. In this article, we extend the aggregation operator into a new way of weighted aggregation operator. The weight of the criterion in this aggregation operator is calculated based on the neutral membership value of the criteria. Weight of the criteria has been change due variation of neutral membership value. We defined different linguistic function related to PFN. Those function can modify or adjust the PFN based on linguistic significant.

Remaining of the article organized is as follows. Some relevant ideas of picture fuzzy sets are recalled in Section 2. Incorporate the range and coefficient of PFS in Section 3 followed by the proposed method in Section 4. Analysis the propose method with a real life example in Section 5. Drawn the final conclusion in Section 6.

2. Related Work

In this section are discussions some significant parts of picture fuzzy set theory and different operation and aggregation operator on picture fuzzy sets over the universal set X. A PFS $A=\left\{\left(x,\mu_A(x),\eta_A(x),\nu_A(x)\right)\middle| x\in X\right\}$ on the set X is a described object where $\mu_A(x)\in[0,1]$ is called the degree of positive membership of x in A, similarly $\eta_A(x)\in[0,1]$ and $\nu_A(x)\in[0,1]$ is called degree of neutral and negative membership of x in A respectively. These three parameters $\mu_A(x),\eta_A(x)$ *and* $\nu_A(x)$ of the picture fuzzy set A satisfy the following condition $\forall x\in X, 0\le\mu_A(x)+\eta_A(x)+\nu_A(x)\le 1$

Then, the degree of refusal membership $\rho_A(x)$ of x in A can be estimated accordingly, $\forall x\in X, \rho_A(x)=1-\left(\mu_A(x)+\eta_A(x)+\nu_A(x)\right)$.

The neutral membership $\eta_A(x)$ of x in A can be consider as degree of positive membership as well as degree of negative membership whereas refusal membership $\rho_A(x)$ can be explain as not care about the system. When, $\forall x\in X, \eta_A(x)=0$, then the PSF reduce into IFS.

The normal PFS can be present in the form $A=\left(\mu_A(x),\eta_A(x),\nu_A(x),\rho_A(x)\right)$ consider as a picture fuzzy number (PFN), where $\mu_A(x)\in[0,1]$, $\eta_A(x)\in[0,1]$, $\nu_A(x)\in[0,1]$, $\rho_A(x)\in[0,1]$ and $\mu_A(x)+\eta_A(x)+\nu_A(x)+\rho_A(x)=1$

Generally, the short representations of PFN is $A=\left(\mu_A(x),\eta_A(x),\nu_A(x)\right)$ and omit $\rho_A(x)$ value from it.

2.1 Operation on PFS

For two PFS A and B, then some operations are defined as follows [19, 30]

I. $A\subseteq B$ $\begin{pmatrix}\forall x\in X, \mu_A(x)\le\mu_B(x), \eta_A(x)\le\eta_B(x),\\ \nu_A(x)\ge\nu_B(x)\end{pmatrix}$

II. $A=B$ If $\left(A\subseteq B \text{ and } B\subseteq A\right)$

III. $A\cup B=\left\{\begin{pmatrix}x,\max\left(\mu_A(x),\mu_B(x)\right),\min\left(\eta_A(x),\eta_B(x)\right),\\ \min\left(\nu_A(x),\nu_B(x)\right)\end{pmatrix}\middle| x\in X\right\}$

IV. $A\cap B=\left\{\begin{pmatrix}x,\min\left(\mu_A(x),\mu_B(x)\right),\\ \min\left(\eta_A(x),\eta_B(x)\right),\\ \max\left(\nu_A(x),\nu_B(x)\right)\end{pmatrix}\middle| x\in X\right\}$

V. $com(A)=\overline{A}=\left\{\left(x,\nu_A(x),\eta_A(x),\mu_A(x)\right)\middle| x\in X\right\}$

2.2 Properties of PFS based operations

According to [31] some properties of PFS of these operations are follows:

I. If $A\subseteq B$ and $B\subseteq C$ then $A\subseteq C$;

II. $\overline{\overline{A}}=A$;

III. Operations $\cup$ and $\cap$ are commutative, associative and distributive;

IV. Operation $\cup$, com and $\cap$ satisfy the De Morgan law.

2.3 Score and Accuracy Function of PFSs

Score and accuracy function are most applicable in real life problem for ranking the PFNs. Regarding this score and accuracy function are introduced by[13], score function S can be defined as $S(A)=\mu_a-\nu_a-\eta_a$ and the accuracy function H is given by $H(A)=\mu_a+\nu_a+\eta_a$ where $S(A)\in[-1,1]$ and $H(A)\in[0,1]$ of the picture fuzzy number $A=\left(\mu_a, \eta_a, \nu_a, \rho_a\right)$.

2.4 Arithmetic operation on PFSs

Let $\alpha=(\mu_\alpha,\eta_\alpha,\nu_\alpha)$ and $\beta=(\mu_\beta,\eta_\beta,\nu_\beta)$ be two picture fuzzy numbers, then

I. $\alpha.\beta=\left((\mu_\alpha+\eta_\alpha)(\mu_\beta+\eta_\beta)-\eta_\alpha\eta_\beta,\eta_\alpha\eta_\beta,1-(1-\nu_\alpha)(1-\nu_\beta)\right)$

II. $\alpha^\lambda=\left((\mu_\alpha+\eta_\alpha)^\lambda-\eta_\alpha{}^\lambda,\eta_\alpha,1-(1-\nu_\alpha)^\lambda\right),\lambda\succ 0$

2.5 Picture fuzzy weighted geometric operators

Let $p_j=(\mu_j,\eta_j,\nu_j,\rho_j)$ (j=1, 2, …n) be a collection of PFNs, then the picture fuzzy weighted geometric(PFWG) operator[] can be obtain as follows:

$$PFWG_w(p_1,p_2,....p_n)=\prod_{j=1}^{n}p_j^{w_j}$$

$$=\left(\prod_{j=1}^{n}(\mu_j+\eta_j)^{w_j}-\prod_{j=1}^{n}\eta_j^{w_j},\prod_{j=1}^{n}\eta_j^{w_j},1-\prod_{j=1}^{n}(1-\nu_j)^{w_j}\right)$$

…. (1)

Where $w=(w_1,w_2,........w_n)$ be the weight vector of p_j (j=1, 2…n) and $w_j>0$ and $\sum_{j=1}^{n}w_j=1$

2.6 Picture fuzzy weighted averaging operators

Let α_j $(j=1,2,.....,n)$ be set of PFNs and let PFWA: $X^n\rightarrow X$ if $PFWA(\alpha_1,\alpha_2,.....\alpha_n)=\omega_1\alpha_1\oplus\omega_2\alpha_2\oplus\cdots\omega_n\alpha_n$ then PFWA is called as picture fuzzy weighted averaging operator and $\omega=(\omega_1,\omega_2,....\omega_n)^T$ be weighted vector of α_j such that $\omega_1\succ 0$ and $\sum_{j=1}^{n}\omega_j=1$

$$PFWA(\alpha_1,\alpha_2,.....\alpha_n)=\omega_1\alpha_1\oplus\omega_2\alpha_2\oplus\cdots\omega_n\alpha_n$$

$$=\left\{\left(1-\prod_{j=i}^{n}(1-\mu)^{\omega_j},\prod_{j=i}^{n}\eta^{\omega_j},\prod_{j=i}^{n}\nu^{\omega_j}\right)\right\} \quad \text{…(2)}$$

3. Range and coefficient of PFSs

The range is the positive quantity of the data set *d* denoted as R(*d*) is defined to be the difference between the largest (max(d)) and smallest (min(d)) observed value in the data set.

R(d)=max(d)-min(d)….. (3)

Example 1: the marks obtained by the ten students of an examination represented by dataset Marks.

Marks = (25, 41, 78, 35, 75, 56, 91, 71, 95, 63).

Max=max(Marks)=95
Min=min(Marks)=25
As per equation (3): R(Marks)=95-25=70

Let P_i $(i=1,2,3,...,n)$ be the set picture sets. Then R(μ), R(η) and R(ν) denote the range of positive, neutral and negative membership degree and estimated following way:

R(μ) = L_μ - S_μ, where $L_\mu=\max_i\{\mu_i\}$ and $S_\mu=\min_i\{\mu_i\}$

R(η) = L_η - S_η, where $L_\eta=\max_i\{\eta_i\}$ and $S_\eta=\min_i\{\eta_i\}$

R(ν) = L_ν - S_ν, where $L_\nu=\max_i\{\nu_i\}$ and $S_\nu=\min_i\{\nu_i\}$

The relative measure corresponding to the range of the data set *d* called the coefficient factor (f) of range is obtained by applying the following formulas:

$$f=(L-S)/(L+S) \text{…….. (4)}$$
$$where\,L=\max(d),S=\min(d)$$

The positive, negative and neutral membership degree of picture fuzzy set have the different affect in decision making methods. Regarding the impact of membership degrees we are consider following three equations for estimating the coefficient factor of three membership degree:

i. positive coefficient factor (f_μ) :

$$(f_\mu)=1+(L_\mu-S_\mu)/(L_\mu+S_\mu) \text{…..(5)}$$

ii. negative coefficient factor (f_ν):

$$(f_\nu)=1-(L_\nu-S_\nu)/(L_\nu+S_\nu) \text{…...(6)}$$

iii. neutral coefficient factor (f_η):

$$(f_\eta)=(L_\eta-S_\eta)/(L_\eta+S_\eta) \text{……(7)}$$

4. Proposed Method/Procedure/Design

Let $A=(A_1,A_2,...........A_m)$ be the set of attributes and $w=(w_1,w_2,...........w_m)$ be the calculated weighted vector of attributes A_i where $w_i\in(0,1)$, $i=1,2,3,.....m$ and $\sum_{i=1}^{m}w_i=1$.

Let $C=(C_1,C_2,..........C_n)$ describe set of alternatives. Suppose that $D=(a_{ij})_{mn}$ be a decision matrix where a_{ij} is the attributes value provided by the expert for the alternative $C_j\in C$ with respect to the attribute $A_i\in A$,

($i = 1, 2, 3.....m$ and $j = 1, 2, 3.....n$) in the form of picture fuzzy number $a_{ij} = (\mu_{ij}, \eta_{ij}, \nu_{ij})$, μ_{ij} indicate the membership value, η_{ij} and ν_{ij} represent neutral and non-membership value respectively. We calculated the weight of the attribute A_i is $w_i = \frac{f_{\eta_i}}{\sum_i f_{\eta_i}}$,where f_{η_i} indicate the coefficient factor of the neutral membership value of the attribute A_i, which is calculated using eqa(7). Then the tradition procedure is as follows:

$$D = \begin{array}{c} \\ C1 \\ C2 \\ . \\ . \\ Cn \end{array} \begin{array}{c} \begin{array}{ccccc} A1 & A2 & ... & .... & Am \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & . & . & a_{1m} \\ a_{21} & a_{22} & . & . & a_{2m} \\ . & . & & & \\ . & . & & & \\ a_{n1} & a_{n2} & . & . & a_{nm} \end{pmatrix} \end{array}$$

Step1: **T**he collective decision matrix $D = (a_{ij})_{mn}$ constructed based on the opinion of the experts using picture fuzzy numbers.

Step2: Determine the largest neutral membership value L_{η_i} and smallest neutral membership value S_{η_i} of all attributes (A_i), $L_{\eta_i} = \max_j \{\eta_{ij}\}$ and $S_{\eta_i} = \min_j \{\eta_{ij}\}$. Then compute the coefficient of neutral degree of each attributes accordingly: $f_{\eta_i} = (L_{\eta_i} - S_{\eta_i}) / (L_{\eta_i} + S_{\eta_i})$

Step3: Calculate the weight of the attributes based on the neutral coefficient factor: $w_i = (1 - f_{\eta_i}) \Big/ \sum_{i=1}^{m} (1 - f_{\eta_i})$

Step4: Based on the collective decision matrix, developed at step 2 and weight of the attributes are computed at step 3, aggregate the attribute values of each alternative into a single aggregated PFN (R_i) for each alternative C_i ($i = 1, 2, 3.....n$) using PFWA operator defined by equation (1).

Step5: Calculated the score value S_i of the overall aggregated PFN (R_i) according score function, defined in section 2.3.

Step6: Rank the alternative C_i ($i = 1, 2, 3.....n$) according to the descending value of the score S_i and hence select the most desirable alternative.

5. Results and Discussion

In this section, we present a numerical example to apply our proposed method and showing its utility in the decision making domain for ranking the students in an examination system with multiple papers. In the examination of graduate in medical science, there are five identical papers like Biochemistry (P1), Genetics (P2), Pathology (P3), Anatomy (P4) and Physiology (P5). For the evaluation, the students are appearing in five separate MCQ based examination for each papers. The final score of the candidate is calculated according to the score of each papers. The score of those papers are present in form of picture fuzzy number. The picture fuzzy number $p_{ij} = \{(\mu_{ij}, \eta_{ij}, \nu_{ij})\}$ indicate the score of i[th] candidate of j[th] paper, here μ_{ij} represent correct part, η_{ij} represent non attempt part and ν_{ij} represent incorrect part of the papers. There is a decision matrix $F = [\mathrm{f}_{ij}]_{9x5}$ is represent in Table 1 showing the score of the paper of the 9 number of students in the form of picture fuzzy number p_{ij} (i=1,2……5 and j=1,2,3…..9). All the students and papers are indexed by S and P for simplification respectively. The following steps are to be performing for ranking the students:

Step1: Constructed the collective decision matrix (D) based on students' performance as on Table 1.

Step 2: Calculate the neutral coefficient of each paper as per previously defined technique. The neutral coefficients are shown in Table 2.

Step 3: Then, calculate the weight of each papers based on individual neutral coefficient and shown in Table 2.

Step 4: Aggregate the performance of the papers of each students and generated the resultant picture fuzzy numbers (PFNs) and represented in Table 3.

Step 5: Calculated the score value of the students based on the resultant PFN and shown in Table 4.

Step 6: Ranking the students according to score value and ranking sequence of the students as follows: S4, S8, S3, S1, S2, S5, S7, S6 and S9.

6. Conclusion and Future Scope

In this article, we have proposed a neutral degree based weighted aggregation operator and apply it for aggregation of PFNs. We have also defined the range and coefficient estimation technique. The real life example is related to evaluating the performance of the students through the MCQ based examination system with multiple papers. The performance score of the papers represented by picture fuzzy number. Then estimate the weight of the paper by the calculating neutral coefficient of the papers. After that, aggregate the performance score of the papers of each students and generate final score based on this. Finally, ranked the students according to the final score. In future, the proposed method can be modified further for managing grade based examination system and assign the priority of the papers dynamically.

Table 1. Picture fuzzy decision matrix D.

Papers Students	P1	P2	P3	P4	P5
S1	(0.62,0.2,0.14)	(0.6,0.2,0.15)	(0.75,0.15,0.05)	(0.7,0.15,0.1)	(0.7,0.1,0.15)
S2	(0.7,0.15,0.1)	(0.75,0.1,0.1)	(0.85,0.1,0.05)	(0.7,0.2,0.06)	(0.65,0.2,0.1)
S3	(0.6,0.2,0.15)	(0.82,0.1,0.08)	(0.72,0.14,0.14)	(0.81,0.11,0.08)	(0.8,0.11,0.09)
S4	(0.7,0.1,0.1)	(0.73,0.15,0.12)	(0.75,0.13,0.12)	(0.85,0.12,0.03)	(0.72,0.13,0.15)
S5	(0.81,0.1,0.07)	(0.76,0.17,0.07)	(0.83,0.15,0.02)	(0.9,0.05,0.05)	(0.76,0.2,0.04)
S6	(0.65,0.15,0.1)	(0.68,0.24,0.08)	(0.65,0.18,0.17)	(0.7,0.17,0.13)	(0.66,0.25,0.09)
S7	(0.55,0.17,0.18)	(0.7,0.2,0.08)	(0.7,0.16,0.07)	(0.65,0.13,0.12)	(0.6,0.1,0.2)
S8	(0.7,0.1,0.1)	(0.73,0.15,0.12)	(0.75,0.13,0.12)	(0.85,0.12,0.03)	(0.72,0.13,0.15)
S9	(0.81,0.1,0.07)	(0.76,0.17,0.07)	(0.83,0.15,0.02)	(0.9,0.05,0.05)	(0.76,0.2,0.04)

Table 2. Neutral coefficient and weight of each papers based on collective matrix D.

Papers Students	P1	P2	P3	P4	P5
Neutral coefficient	0.33	0.59	0.29	0.6	0.43
Weight	0.24	0.15	0.25	0.14	0.21

Table 3. Aggregation PFN of the students

Students	S1	S2	S3	S4	S5	S6	S7	S8	S9
Resultant PFN	(0.81,0.1,0.07)	(0.76,0.17,0.07)	(0.83,0.15,0.02)	(0.9,0.05,0.05)	(0.76,0.2,0.04)	(0.73,0.15,0.12)	(0.75,0.13,0.12)	(0.85,0.12,0.03)	(0.72,0.13,0.15)

Table 4. Final score of students

Students	S1	S2	S3	S4	S5	S6	S7	S8	S9
Score	(0.64)	(0.52)	(0.66)	(0.8)	(0.52)	(0.46)	(0.5)	(0.7)	(0.44)

Data Availability

The relevant data analysis of this article are included within the text. No other hide information as well as complex analysis has not attached with it.

Conflict of Interest

The author declare no conflict of interest.

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Authors' Contributions

All the authors has been perform all activities and equally contributed to develop different parts such as literature study, concept implement, drafting the article, data analysis and finalization of the article.

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